Dialectics of Nature*)

——*On quantum mechanics*——

Mituo TAKETANI

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On mathematization

In the present day, cries of crisis have risen in every field, whether it is social or cultural. The snake should grow out of its aged skin. We must understand Goethe's *Stirb und Werde*—the negation of the negation. Even in mathematics and physics, that were considered to be strictly confined into formal thought, there have arisen, since the end of the last century, such contradictions that could never be veiled. What is known as the *formale Verstandesdenken* has turned out in a determined way not to be able to go abreast with mathematics and physics. These have learned such various facts from Nature that could be hardly veiled by the formal thought. Physics has developed so much that the formal thought could never understand it. There has arisen in consequence the so-called mathematization which insists that "equation is everything". It has been promoted by general social unrest to become a subject of empiricism, Machism, scepticism, symbolism, agnosticism, and so on. Hitherto, it has been emphasized that the method of physics should be that of formal logic or analytical logic. This is nothing but an abstract understanding of physics. Even mathematics is a rich reflection of dialectics of Nature. It can, therefore, penetrate well into inner structure of Nature, which is out of consideration in the *Verstandesdenken.* The things reflected by mathematical method thus have realistic meaning. It never hold that "equation is everything". It is merely the failure on the part of those philosophies to understand the meaning of equation, that produces their interpretation of equation as symbol. Physics is frequently looked upon as mere treatment of functional relations, and we know that cursory imitative attempts have been stimulated in the fields of biology and economics by the outstanding results achieved in mathematical natural science. All these things rest

*) Contributed to the March issue of the journal *Sekai-Bunka* (World Culture) in 1936, under the pen name of Kazuo Tani.**)

**) Editor's note: This pen name was forced to be used in order to avoid pursuits of the Police of Japanese Militarism. He was also forced to be restricted in his way of describing the matter concerned here because of "the political reasons in the oppressive epoch". In spite of such caution, he was arrested for the first time by the Police of Japanese Militarism on September 13, 1938, on account of this article and of the following article, "A New Stage in the Development of Modern Physics", and was restrained until April 22, 1939.

on the basis of superficial understanding of physics. It is in deeper dialectical comprehension of essence and phenomenon that every achievement of physics exists. Before formulating equations, it is required to know the substantial structure of object, in the sense that what things there are and under what kinds of interaction they are, i.e., the model of object.*) Physics is not a mere phenomenology. This holds true with classical mechanics as well as quantum mechanics. In classical mechanics, the image given by the model has its meaning as such. In quantum mechanics, however, the mechanical image given by the model alone does not suffice, but is considered in the sense of the moment. And so, from the model we construct the Hamiltonian, with which the equation is derived at last. This is one of the meanings that Bohr's correspondence principle implies.

The interconnection between essence and phenomenon leads us directly to the comprehension of that between observation and recognition. *Das Wesen erscheint, und die Erscheinung ist wesentlich.* Without observation, recognition cannot be obtained. But, observation itself is not recognition. From the identification of these two facts, there arise confusions. Heisenberg's principle of uncertainty indicates the limit in each observation, but never the limit of recognition. Recognition is the copying of Nature. It is however not a dead static reflection like an image in a mirror. On the contrary, our recognition penetrates deep into, and still deeper into, the essence of Nature. And, from the historical point of view, it is the process of bringing the copy into agreement with the object, too. It is by no means arbitrary "production" of the subject.

Dialectics in the foundation of mathematics

Mathematics is, at every point, accomplished by dialectics. This fact is nothing but what has been concealed by formal thought. I would like to show this fact by taking an example from Engels' *Anti-Dühring*, which gave birth to various oppositions and also is one of the hottest topics of discussion in our country,**) while at the same time giving my solution that seems appropriate. According to Engels,***) "Let us take any algebraical magnitude whatever: for example, a. If this is negated, we get $-a$ (minus a). If we negate that negation, by multiplying $-a$ by $-a$, we get a^2, i.e., the original positive magnitude, but at a higher degree, raised to its second

*) Author's note in compilation (1946): Similar thing was pointed out by Schrödinger at the same time as this article was written. He used the word model in nearly the same meaning, but his theory ended in a version of ordinary empiricism on account of his failure to understand dialectics. (E. Schrödinger, *Naturwissenschaften* **48** (1935), 807, 823 and 844.)

**) Author's note in compilation (1946): I meant principally those in the journal *Yuibutsuron Kenkyu* (Studies in Materialism).

***) Translator's note: This English version is quoated from F. Engels, *Anti-Dühring* (Foreign Languages Publishing House, Moscow, 1947).

power. In this case also it makes no difference that we can reach the same a^2 by multiplying the positive a by itself, thus also getting a^2. For the negated negation is so securely entrenched in a^2 that the latter always has two square roots, namely a and $-a$." It seems fairly difficult and somewhat unnatural to consider this example as one of the negation of the negation, so that we could develope arguments in the sense of the *Verstand* either for or against it. For instance, such an objection that the negation of a is $-x$ due to the freedom of the negation,*) is a formal negative judgement, which asserts non A as a formal negation of A. As was already criticized by Hegel, this is irrelevant. The negation of A should be, in a concrete dialectic, what represents the real contradiction as the moment of development of A. Even without the procedure of negating twice, a^2 could be obtained by a single affirmation or negation (multiplying by a); Whether would the negation mean a multiplication by negative number, or a mere multiplication?; "Positive number $\times$ negative number = negative number, negative number $\times$ negative number = positive number"—this rule written down in verse would be nothing but the example of a^2 in *Anti-*D*Dühring*; This would not be the case because for $a<0$, $a \to -a \to a^2$, $a^2>0$, so that it would be reduced to the rule "negative number $\times$ negative number = positive number, postive number $\times$ positive number = positive number", not giving in any way the return-to-itsef; Which would it then be, verse or prose?; and so on—From such refutations, one can hardly conclude at once that there is no dialectic in elementary mathematics, or no dialectic in equations while there are in mathematics, if one should not be hasty and idle. On the other hand, no step could be advanced if one would advocate and force *parteilich* dialectics on others, without lending his ear to those refutations of the sense of the *Verstand*.

However, the main point of dialectics is "the unification of antagonism", and the other laws are nothing but its manifestations. From this, considering more fundamentally, we see that a is dialectically in antagonism to $-a$, and the existence of a is the condition for that of $-a$. By being squared, they are unified to a^2. In other words, a^2 is $a \times a$ as well as $(-a) \times (-a)$. These two things contradicting each other are unified to a^2. Therefore, the law of identity that asserts "A is A but is not non-A" in formal logic does not hold in this case. That is, we have $a^2 = a \times a = (-a) \times (-a)$. The sign of equalty in mathematics has nothing to do with the law of identity in formal logic, but reflects the dialectic of difference and identity. It is generally said that formal logic is abstract. This is so, because, speaking more strictly, formal logic holds only for a set of such elements that could be considered, in contrast to any case with concrete things, separately from

*) Author's note in compilation (1946): Prof. Hajime Tanabe's**) viewpoint.

**) Editor's note: A leading philosopher of the academism in Japan at that time.

each other. Once they are taken in relation with each other, it does not hold any more. It does not hold for groups, field, rings, and so on. Suppose, for example, $BA=C$ and $DA=E$ in a group. We have then $A=B^{-1}C$ as well as $A=D^{-1}E$. The sign of equalty in mathematics never indicates the law of identity in formal logic, and rather represents a transformation. It is the interconnection, and reflects a good lot of dialectics of one thing into another, transition or flow into other, one and many, and so on. In this way, transformation is fundamental in mathematics. The more physics reflects Nature concretely, the more use is made of transformation extensively in physics. This is to reflect Nature in its interconnection, in its flow. It is in this meaning that use is made of tensors in the theory of relativity, and of matrices and operators in quantum mechanics. By their introduction, various physical quantities have now received deep significance. In classical mechanics, a physical quantity has its meaning at its value, and a physical law has its meaning as a relation between values. Quantum mechanics has overthrown classical mechanics fundamentally. However, the laws thus obtained are provided with the same forms as those in classical mechanics. The laws in quantum mechanics turn out, however, to be the relations among physical quantities themselves, rather than those among their values. That is, in classical mechanical laws, emphasis is laid more or less on the side of quantative relation, whereas in the quantum mechanical ones, it is laid on the side of qualitative relation. This is also one of the excellent proofs of the dialectic of development. On the contrary, mathematization or formalization of physics with tensor, matrix, etc., being put opposite to differentiation, in the so-called logic of higher dimensional directional quantities,*) is an abstract viewpoint of seeing the outside rather than the contents. Differentiation is also one kind of transformation. It transforms a vector $f(x)$ in an infinite dimensional space into $f'(x)$, where $f'(x)=Pf(x)$ with $P=d/dx$. Upon being so renewedly seen, differentiation obtains significant meaning in quantum mechanics.

In such a way, groups, etc., reflect dialectics. They could not, however, substitute dialectics. In recent times, there are some minor philosophers**) who introduce groups, etc., together with the supposition that logic is exhausted by them, in order to cover their repudiation of dialectics even under the requirement of introducing some kind of dialectics. Physics rather reflects dialectics far richly than groups, etc., than the mathematics used in it. The supposition, done by minor philosophers on being dazzled by the word "operator", of the operating subject under whose operation every thing is considered, is just as absurd as the supposition of the negating subject from the word "negation" in dialectics.

*) Author's note in compilation (1946): Viewpoint of the Tanabe's school.
**) Author's note in compilation (1946): View of the Tanabe's school.

In the theory of set, each element is treated rather unconnectedly with others. That is, in the theory of set, formal thought plays an important role, whose contradiction is exposed at infinity. As has been pointed out by Mr. Yūichi Seta*) in his article**) (*Yuibutsuron Kenkyu* ((Studies of materialism)), March, 1935) which is the first valuable one in this field of research, it is obvious that such contradiction is in the form of the *schlechte Unendlichkeit* as well as of the moment.

Quantum mechanics and dialectics

In quantum mechanics, the two contradicting phenomenal forms of wave and particle—the two images that are excluded from each another in the *Verstand* which is historically as old as the study itself, is grasped through their unification into the essential concept of state. For a physical system to be limited spacially, it is required to produce a wave-packet by superposing a number of waves with different wave-length. This is "the principle of superposition" that is fundamental in quantum mechanics. The system limited in space thus contains in itself a contradiction which gives rise to its self-movement—the spreading-out of the wave-packet. In other words, the momentum of the system becomes uncertain within the limit that states with different wave-length, i.e., with different momentum, are superposed in limiting the system spacially. This is the so-called principle of uncertainty due to Heisenberg. When a state is formed from a number of states by superposition, *Unbestimmtheit* prevails among the respective states so superposed. In observation, there arises the reduction of state through which the *Bestimmtheit* becomes prevailing for the first time. To which state it is reduced through the reduction can be predicted in this case, however, only in the sense of probability. That is, there are uncontrollable interactions acting in the observation. This probability is of a character quite different from that of classical one. Classical probability implies *bestimmt und unbekannt*, while in this case, what is *unbestimmt* in its nature becomes to be *bestimmt* by observation. In classical probability, everything is *bestimmt*, and what is *unbekannt* becomes *bekannt* by observation.***) Suppose, for example, an electron or a photon passing through a screen with two slits on it. According to classical theory, so long as it is a particle, it passes through either one or the other of these two slits. Although without observation it is *unbekannt*

*) Translator's note: Yūichi Seta is the pen name of Hiroo Mita that he was forced to use in order to avoid pursuits of the Police of Japanese Militarism.

**) Author's note in compilation (1946): In this article of the history of mathematics, Mr. Mita has shown splendidly that the above mentioned example given by Engels is just for "the negation of the negation" by analyzing historically the origin of negative numbers.

***) Author's note in compilation (1946): According to the terminology of Pauli, *Handbuch der Physik*, Vol. 24 (Springer, Berlin, 1933).

which slit it passes through, it is *bestimmt* that the particle passes through either one of them. Whereas, in quantum mechanics, one particle as itself should pass through both of the two slits at the same time, because of the interference which takes place after its passing through the screen. The interference does not occur if it is determined by observation: which slit the particle is passing through. Which slit the particle has passed through, is, therefore, *unbestimmt* in its nature, in order to give rise to the interference. Like this, the superposition of states is fundamentally different from that of probabilities.*) In other words, wave function Ψ that represents state has a meaning more profound than probability. Wave function is essential. That is, laws in quantum mechanics are not statistical ones. Probability comes out in observation. It is a phenomenal form. Since only by the abstract process of $\Psi \rightarrow |\Psi|^2$, the wave function is bestowed a meaning in the sense of probability, statistical laws are nothing but the phenomenological descriptions of observation. In doing so, abstraction is made of the phase factor which plays a role in interference.—This means the fact that interference would not occur if we observe through which slit the particle passes.

For a closed system, the motion of a state obeys the causality in the strict sense. But in an observation, it behaves unpredictablly on account of uncontrollable interactions, so that the results of the observation can be predicted only statistically in the sense of probability. This allows the cry that "The causality is denied". In the kinetic theory of gas, motion of each molecule is *unbekannt* but is *bestimmt*, and obeys the causality. Thermal phenomena in a mass of molecules are however statistical. In this case, coordinates indicating position and momentum of each molecules are known as hidden parameters. On the analogy of this, there are risen the following two kinds of opinion that "In quantum mechanics, too, there should exist such parameters hidden behind its probability, and then,

(1) since these parameters are not to be known by us, they would be the wills of electrons, or would be in the region of god, or

(2) quantum mechanics is incomplete, so that these parameter should be found and introduced, to make it a complete theory".

The former is a sort of agnosticism, while the latter is a thinking of finding no rest without the mechanical causality. Although we cannot say that it will still be utterly impossible to introduce something corresponding to such parameters into quantum mechanics, even if quantum mechanics changes completely in its feature in the future development of physics, it is quite meaningless, and is impossible on account of the structure of quantum

*) Author's note in compilation (1946): This point was misunderstood by Prof. Tanabe.

mechanics,*) to introduce such parameters into it. Quantum mechanical physicists will perhaps repeat the following words of Laplace that "His majesty, I do not need such a hypothesis". In classical mechanics, the absolute motion in reference to ether was a kind of hidden parameter. Every effort was made in search of it, but ended in failure. It was at this moment that the theory of relativity came out to discard this objectively meaningless parameter, thus representing the world as the one in which only the relative motion is meaningful. This was the first time that the time, space, matter and motion were grasped in close dialectical interconnections. We can hold the prospect that any effort in the future will not find the absolute motion. Quantum mechanics is full of contradictions which should be *aufgehoben* in future. In quantum mechanics, there is however no such contradiction seen at present that would lead to its development in the direction of hidden parameters. We have no need to expect and require such parameters. Quantum theory sets forth itself as the one that can be complete without these parameters. As will be shown in the following analyses, the lack of such parameters by itself signifies its dialectical interconnection. To require such parameters is nothing but to deteriorate the necessity to the contingency.

In quantum mechanics, when one system is composed out of two systems, there appears in the whole what is more than the mere sum of respective components. This is the relation between the part and whole, which cannot be understood with the formal logic. In the whole thus composed, each component is completely different from what it is when put independently. This does not consist in a mere mechanical interaction, but does in the interpenetration of the space itself. It is a close dialectical connection in the composition of the whole out of its parts. (It should be particularly noted that in a composition out of two identical systems, two electrons for example, each one loses its individuality completely.) And, the whole system as a

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*) Author's note in compilation (1946): This was proved by Neumann; J. v. Neumann, *Mathematische Grundlagen der Quantenmechanik*, (Springer, Berlin, 1932). Neumann's book proved that there is no contradiction included in quantum mechanics. I owe much to Neumann's book in my analysis given here. However, Neumann's theory does not imply any concrete observation, I think. Schrödinger's articles which I quoated in my note on page 28 are also concerned with this point. Bohr's reply in Bohr-Einstein's dispute could be said to be based on a viewpoint more advanced than previous ones in this regard. Opinions in Pauli's book could also be said not to be satisfactory in this connection.

On this point, I made my opinion public at *Gakujutsu Kenkyu Kaigi* (Council for Scientific Research) in October, 1942.**) In it, I pointed out that the cut between observer and object in actual observation is, in contrast to Neumann's thought, determined objectively, and lies at the place where microscopic phenomena is transmitted into macroscopic phenomena, that is, in the observational instrument, being irrelevant to the human subject. Otherwise, we would fall into the difficulty pointed out by Schrödinger. I shall publish on another occasion a more detailed account of this point.

**) Editor's note: cf. the following article, "Observation Problem of Quantum Mechanics", by Taketani.

closed system obeys strictly the causality. The observation is to construct a composite system out of the observer (not the subject) and the object. The composite system as a whole is causal in its strict meaning. However, the observation is not concerned with this composition only. To be content only with it*) is a kind of mechanism, or an abstract viewpoint not seeing things concretely. The observation is to extract, from the whole, the object, that is, one part, with the setting up of a cut. In this case, the laws of identity and of sufficient reason do not hold for the part, since it is extracted from the whole which is more than the mere sum of the parts. As a result, there takes place the reduction of state. That is, one and the same state *erscheint* differently in the observation, and the laws of identity and of sufficient reason do not hold because the *Erscheinung* can be predicted only for its probability.—This is the true content of the so-called denial of the causality.—It is the process of the self-realization in which the essence *erscheint* in its whole interconnection. The conjugation of necessity and contingency *erscheint* as the probability. The uniform equality, dependent only on the spacial extent of the state to be reduced into, of pure contingency, i.e., of the probability, is conversely assumed as a necessity, and has its base on the law of the composition. Then, should we despair of getting any image of the existence on account of the limit in observation, or other reasons? No, certainly not! A single observation is contingent, but observations in total can reflect it as completely as possible (by making a large number of observations).—By completely it is meant without omission but not correctly. The correctness involves historical restrictions.—A sum of observations in the sense of collecting up simply, constitutes statistical as well as phenomenological laws of describing phenomena, but human thought penetrates deeper into phenomena, thus arriving at essential as well as necessitarian laws. Schrödinger's equation, for example, has such a meaning. It does not mean the denial of the existential significance of the concept of state, that this concept cannot be bestowed with any image. This is the very evidence that we have learned the law, and learned the logic, from the external world. Every time we penetrate deeper into Nature, we encounter with something *fremd* to our previous images. The repetition of practice is called for, before we feel them as our own belongings. The essential concept thus obtained by thought is shown to be a correct reflection of the object by means of practices. The state, which is essential and represented by a wave function, *ersheint* in the forms of wave (interference), particle and probability. The so-called non-causal reduction of state in observation represents neither "the action on the part of the subject" nor "the denial of the causality", but is based upon the dialectic of the law of matter, i.e., of the law of composition in quantum mechanics. There is not any

*) Author's note in compilation (1946): View of the Tanabe's school.

ground for supporting such a view as to ascribe the principle of uncertainty to the operations on the part of the subject.*) Why is there an action beyond our control, when the subject is in operation? Such an abstract view also does not provide any basis for any concrete feature of the principle of uncertainty. It ignores the existence of observation in which the reduction of state does not take place. Subjectivity is not such a mystical thing, but is a practice based upon an objective necessity. The fact that actions in observation are based upon the law of matter is best understood from the fact that one and the same result is obtained irrespective of the location of the cut between the observer and the object.**) This is not the identification of the subject and the object, as is clear from the discussion given above.**) The relation between the causality and statistical laws in quantum mechanics is very much analogous to that in the kinetic theory of gas. In the kinetic theory of gas, the law of the phenomenon as a whole is statistical, being founded by the hidden parameters of each particle that obeys the causality in the strict sense. Quantum mechanics is statistical in the part, that is, in the phenomenon, being founded by the state of the system as a whole that obeys the causality in the strict sense. The statistical laws of the phenomenon in both these cases are founded by the necessitarian causal laws, though their directions are opposite each other.

As is given above, in quantum mechanics the dialectics of the unification of antagonism, the essence and phenomenon, the part and whole, as well as the necessity and contingency, are closely interconnected to each other. Quantum mechanics is showing remarkable difficulties and contradictions in the new fields of atomic nuclei, high energy phenomena, negative energy states, and particularly in quantum electrodynamics. Those which will rescue them, are only dialectical analyses as well as dialectical experiments.

We have to distinguish the dialectic that is grasped concretely as the *aufgehobt* contradiction, with the contradiction that is caught in the sense of the moment as the abstract reflection. The former takes the form of the *wahre Unendlichkeit*. For example, the light velocity in the theory of relativity, Planck's constant, and the principle of uncertainty, are of the form of the *wahre Unendlichkeit*, as a kind of atomism opposing to the partition of phase-space in the sense of the *schlechte Unendlichkeit*. In his recent paper, Born has introduced such atomism as the principle of finiteness. On the other hand, the latter is the contradiction in the sense of the moment as well as the *schlechte Unendlichkeit*. For example, the zero-radius of electron, the infinite self-energy, the infinite density of Dirac's negative energy states,

*) Author's note in compilation (1946): View of the Tanabe's school.

**) Author's note in compilation (1946): See my note on page 33.

and so on, are central difficulties in modern quantum mechanics,*) and are clearly in certain interconnections each other. It is the task of natural science to make what is *momentlich und wahre unendlich* to proceed to what is concrete, so as to grasp it. Must be pointed out by dialectical analyses the correct direction, along which contradictions caught as moment should be *aufgehoben*, so as to find out interconnections among various forms. Natural science is thus to reflect dialectics of Nature concretely in the extent that they stand, and is never non-dialectification**) of dialectics. Dialectics cannot be represented by anything other than dialectics. One who supposes that it is nothing but dialectic to see contradictions in the form of the *Moment und Unendlichkeit*, could not get rid of the criticism of being engaged in fantastic dialectic, too. From such a viewpoint, one is at once led to the conclusion that in actual practice—in technics—there is no dialectic, but there is non-dialectical and analytical logical method only. Dialectic is not to mystificate antagonism under the pretense of getting its solution, giving for it the exclamation "How it is dialectical!" remote from it, but is to grasp antagonism in its unification concretely and definitely. One misunderstands non-dialectical analytical logic for dialectic involved in actual analysis, in saying that "Reflecting dialectics at its foundation, physics should at the same time be concerned with the formation of an analytical theory, with the help of experiments and mathematics of its own. Being seen from this side, physics is still analytical logical rather than dialectical. — Just as mathematics is to be said as the product of technical thought which non-dialectificates the dialectical, so physics should have the side of non-dialectificating dialectical existence".**) Recognition is never gained with analyses of such kind. Such dialectic that is separated from and is put in the sense of opposing actual sciences is fantastic dialectic. Such philosophy of Nature thus goes back to classical Greece—without understanding the historical negation, rushing back into classical Greece, into its natural philosophy, as if giving no ear to Dante's sorrowful cry: "*Non vi si pensa quanto sangue costa.*" ***) ("No one thinks about how much blood it has cost.")****)

*) Author's note in compilation (1946): According to developments in the meson theory, it turns out more and more clear that these points are great central problems to be solved for the forthcoming new theory.

**) Author's note in compilation (1946): View of the Tanabe's school.

***) Translator's note: Dante, "*Divina Comedia*", *Paradiso, Canto* XXIX, *Linea* 91.

****) Editor's note: This Taketani's article was once published in Englsh, on being translated by Osamu Kuno and Arata Ishimoto, in the journal, *The Science of Thought* No. 2 (1956), p. 40.